CD sundials

PAPER

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Abstract

In this paper I present various equatorial sundial designs that use diffraction on a CD to display time. The designs described in the paper include a transparent CD sundial equipped with a small terrestrial globe. This sundial is compact, pedagogically useful, can be used throughout the year, and provides the user with a wealth of information.

1. The equatorial sundial

Sundials have a history of at least three and a half thousand years. Their basic principle is brilliantly simple. The Sun changes its position in the sky from morning till evening, and since this (seeming) motion repeats itself in a reliable and predictable way, it can be used to measure time. The majority of sundials are based on light and shadow: a stick called the *gnomon* casts its shadow on a flat plane called the *dial*, equipped with a suitably calibrated scale, and, using this shadow as an 'hour hand', we can read the current time on the scale. Considering the geometry of our built environment, it is not surprising that sundials in parks and gardens usually have horizontal dials, while those on the walls of palaces or churches have vertical dials. The conceptually simplest geometry, however, is given by the *equatorial* configuration: here the plane of the calibrated dial is neither horizontal nor vertical, but is oriented parallel to the plane of the Equator. The plane of an equatorial sundial is vertical at the Equator, horizontal at the poles, and makes an angle of $90° - 47.5° = 42.5°$ in a city such as Budapest that lies at a Northern latitude of 47.5°. Figure 1 schematically shows an equatorial sundial on the Northern hemisphere during summer. (Throughout the article I assume that the user of the sundial lives on the Northern hemisphere. I leave it to the motivated reader to use the basic design principles outlined below to adapt them for CD sundials that operate on the Southern hemisphere.)

The calibrated dial that the user is holding in her hand and is parallel to the plane of the Equator is shown as a grey line. The gnomon (a little black arrow in the figure) is perpendicular to the dial, i.e. parallel to the rotational axis of Earth, and points towards the Pole Star. As seen in the figure, the gnomon casts a shadow on the top of the dial, so you can read the time by looking at the dial from above. Since Earth rotates through an angle of 15° every hour, lines of hour divisions must be drawn on the dial at regular 15° intervals. (Note that this is slightly complicated by two facts: (1) Earth follows an elliptical, rather than circular, orbit around Sun, and hence its speed relative to Sun is not constant; and (2) Earth's rotational axis is not perpendicular to the plane of its elliptical orbit, but makes an angle of 23.5° with the perpendicular. I will discuss these two phenomena and their effects on sundial design in more detail later.)

Figure 2 shows the same sundial 6 months later, in the winter. Note that for simplicity the figure shows the user holding the sundial in her hand again, but in practice we can imagine that the sundial was simply placed at some location with the proper orientation shown in figure 1, and now we look at it again after 6 months. In figure 2, the gnomon now casts its shadow on the *bottom* of the dial. The transition between the shadow coming from the top and the shadow coming from the bottom occurs on the day of the Autumn Equinox. The opposite transition occurs on the day of the Spring Equinox. On these two days sunlight arrives at the dial at a very shallow angle.

The fact that the gnomon casts its shadow on the bottom of the dial may cause problems when using the sundial, since it is often difficult or uncomfortable to look at the dial from below. One solution to this problem is to make the dial out of a transparent material. Such a sundial, which I designed for children at Kicsi Bocs Óvoda ('Little Cubs' Kindergarten) in Budapest, is shown on photo 3.

This is a simple equatorial sundial, with a pencil acting as the gnomon and a dial made of paper. The paper dial is translucent enough so that the shadow of the pencil is clearly visible even in the winter, when the shadow is cast from below, and the dial is observed from above. This kind of sundial can be used throughout the year, and so it should display normal time (used between September and March) as well as daylight saving time (DST) (used in many countries between March and September). On the sundial shown in photo 3, normal time markings are printed in black and normal time is denoted with a 'snowflake' symbol, while DST markings are printed in grey and DST is denoted with a 'Sun' symbol. (The meaning of the Hungarian words on the dial are: 'délután' = afternoon, 'délelőtt' = before noon, 'észak' = North.)

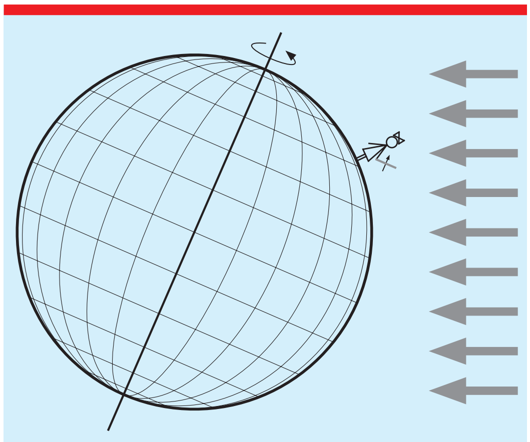

Figure 1. Equatorial sundial on the Northern hemisphere during summer.

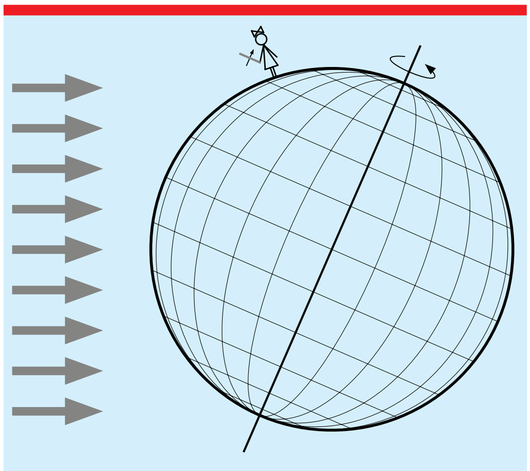

Figure 2. Equatorial sundial on the Northern hemisphere during winter.

2. Equatorial CD sundial

In the remaining part of this paper I present several versions of an equatorial sundial that operates using the principle of diffraction on an optical grating with circular grooves [1], rather than on the principle of a gnomon casting its shadow on a surface. Nowadays CD's and DVD's are used less and less for data storage. As reflection gratings, however, they have nice applications in teaching and demonstrating optical diffraction. The grating grooves are concentric circles, and the distance between adjacent circles—the so-called grating constant—is 1600 nm and 740 nm for CD's and DVD's, respectively. Both values are close to, but above the wavelength range of visible light (~400–700 nm), and hence these disks are ideally suited to demonstrate the color dispersion of optical gratings when illuminated with white light. What is even more important for us now is that, due to the concentric circular geometry of the grating lines, the diffraction of a distant point-like white source appears as a *sharp, radial line* on the surface of the CD or DVD. The *CD sundial* uses this sharp, radial line as the 'hour hand' on a calibrated scale. Photo 4 shows an equatorial CD sundial showing a time of just after 3:30 pm. (This sundial was designed for the same kindergarten as the one on photo 3.) The geometrical configuration of this sundial is very similar to the more traditional one shown on photo 3: the dial,

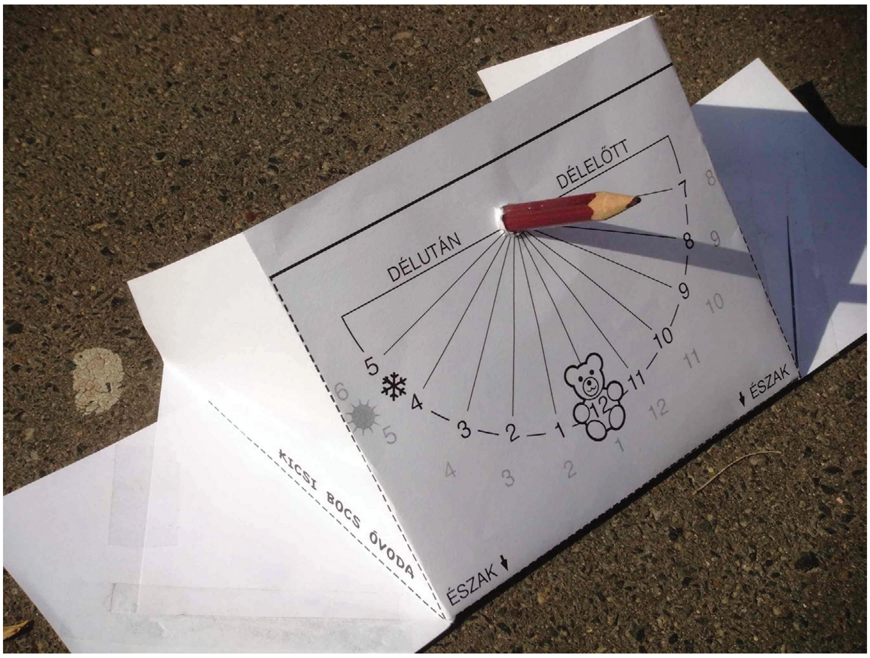

Photo 3. Equatorial sundial with a translucent dial that can be used throughout the year.

Photo 4. Equatorial CD sundial that can be used in the summer.

which is the CD itself, is parallel to the plane of the Equator, and the 'gnomon', which in this case is the imaginary straight line connecting the center of the CD with the viewer's eye, points toward the Pole Star. In the traditional version (photo 3) special care must be taken to set the gnomon exactly perpendicularly to the plane of the dial. The equivalent requirement for the CD sundial is that the viewer must *look at the CD exactly perpendicularly*. This can be fulfilled easily. The shiny surface of the CD acts as a mirror, so all the viewer has to do is to make sure that, while keeping one of her eyes shut, she sees her face reflected in the surface of the CD, with the image of her open eye at the center of the CD.

The CD sundial in photo 4 only has hour-markings for DST. The reason is that since the CD is not transparent, this kind of CD sundial can only be used betwen March 21 and September 23, when Sun illuminates it from above.

3. Local time and zone time

As seen in photo 3, the 12 o'clock marking (the 1 p.m. marking during DST) faces North, which means that the shadow of the pencil hits this number when Sun shines exactly from the South. The same can be seen on photo 4, on the CD sundial that is only calibrated for DST. The 1 p.m. marking (which corresponds to 12 o'clock in normal, winter time) is written at the top of the dial, thus the sharp, bright diffraction line points at this number when sunlight comes exactly from the South. We expect this, since 12 o'clock noon is supposed to be the moment when Sun reaches its highest position in the sky during the day, i.e. when the position of Sun is to the South. This 12 o'clock, however, is what is called *local noon*. The concept of local noon is easy to understand, but is of relatively little use in our daily life. To understand why, consider that the city of Nyíregyháza, close to the Eastern border of Hungary reaches the same position relative to Sun 11 min earlier than Budapest does, and so local noon in Budapest occurs 11 min later than in Nyíregyháza. The city of Szombathely is close to the Western border of Hungary, local noon there occurs 10 min later than it does in Budapest.

Before the mid-19th century, when transportation was relatively slow, this discrepancy between local noons did not pose a problem. Each city and each village organized its everyday life according to its *local time*, calibrated to its local noon. The appearance of trains brought about a great change. In order to avoid confusion and accidents, it was essential to replace the variety of local times with a unified *standard time*, the so-called *railway time*. That way passengers did not have to adjust their watches at every station, and it was clear to everyone that any arrival or departure time printed in the official timetables was given in railway time. In our age, when transportation even between continents takes only a few hours, and communication between any two points on the globe takes only a fraction of a second, the standard times of different countries located around the same longitude are synchronized and called the *zone time*. Sun 'moves' according to local time, but all time-related aspects of our everyday life are governed by zone time.

Naturally, we expect a sundial to show zone time, which is a much more useful piece of information for a watch or a clock to display than local time is. To achieve this we have to *rotate* the calibrated scale on the dial. For example, Budapest is located at a longitude of 19°E. If it were located at exactly 15°E, local noon in Budapest would occur exactly 1 h earlier than local noon at Greenwich (which is located at a longitude of 0°). Since Budapest is located at 19°E, local noon there occurs *still earlier*, by $(19° - 15°) \cdot (60\,\text{min}/15°) = 16\,\text{min}$. This means that exactly one hour before 12 o'clock Greenwich mean time (GMT), when the wristwatch of every person in Budapest (and in Hungary, in general) shows 12 o'clock zone time, and when we want our sundial to show '12 o'clock' too, Sun is *not* at the exact Southern position any more (where it was 16 min earlier), but has already moved slightly towards the West. Thus the hour-markings on our calibrated sundial, if we want to use it in Budapest, must be rotated away from the uncorrected version shown on photo 4, *clockwise*, toward larger (*later*) values. The angle of rotation must be $19° - 15° = 4°$. A CD sundial with such a corrected hour scale is shown on photo 5. The photo was taken at 11:05 am. On this sundial I drew the hour-markings on the CD itself. The short lines along the perimeter are the original markings that correspond to local time, and the longer lines with the numbers written below them correspond to the corrected, rotated scale that displays zone time.

4. Time equation

We may think that our task is completed: we have made an exact equatorial CD sundial. We have set the tilt angle of the CD to match the latitude of our location, and rotated the hour-scale on the dial to match the longitude of our location. Thus our sundial should tell perfectly accurate time, within a precision of ~ ± 3 min, corresponding to the width of the sharp radial diffraction line, on

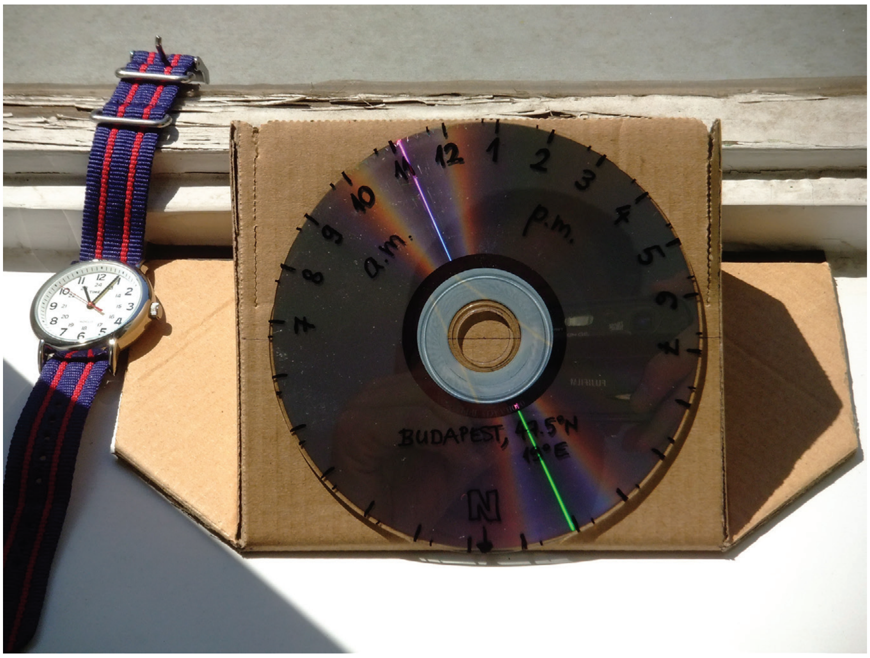

Photo 5. CD sundial with hour scale corrected for zone time.

any sunny day between the end of March and the end of September...

But does it? Well, it would, *if* the orbital speed of Earth around Sun were constant, and *if* the rotational axis of Earth were perpendicular to the plane of the Earth's orbit. If these two conditions were met then *the velocity of our sundial (at rest on Earth's surface) relative to Sun*, i.e. the vectorial sum (1) of the orbital velocity of Earth around Sun and (2) of the orbital velocity of the sundial around Earth's axis, would have the same magnitude in a given hour every day of the year. This, however, is not so, for two reasons. First, Earth follows an elliptical orbit around Sun, thus its orbital speed is not constant: it moves faster in the winter when it's closer to Sun, and more slowly in the summer when it's farther away from Sun. Second, Earth's rotational axis is not perpendicular to the orbital plane, but differs from the perpendicular by 23.5°. This means that, at a fixed time of the day, Earth's orbital velocity vector relative to Sun and the sundial's rotational velocity vector relative to the center of Earth make different angles at different times of the year. Take, for example, 12 o'clock local noon, as the time of the day when we measure the angle between the two velocity vectors. Consider four special days: Summer and Winter Solstice, and Spring and Autumn Equinox. On the days of Summer Solstice and Winter Solstice the two velocity vectors are exactly anti-parallel, and on the days of Spring Equinox and Autumn Equinox they make an angle of $180° - 23.5° = 156.5°$.

As a result of the two effects described above, the first occuring with a one-year periodicity and the second with an approximately half-year periodicity, the relative velocity between Sun and an object at rest on Earth *fluctuates* during the year, and thus in a given location the moment when Sun passes the Southern position on the sky does not occur precisely with a periodicity of 24 h. Detailed calculations lead to the so-called *time equation* [2] which is plotted in figure 6. The horizontal axis shows the months, and the vertical axis shows the correction value that we need to subtract from the time displayed on our sundial (of the type shown on photo 5), in order to tell *exact* zone time. I highlighted the portion of the curve between the Spring and Autumn Equinoxes with a thick continuous line: for the non-transparent CD sundial, which can only be used between 21 March and 23 September, only this portion of the time equation graph is relevant.

The example of photo 5 illustrates this point. This photo was taken on 19 May, and the bright diffraction line on the dial points to approximately

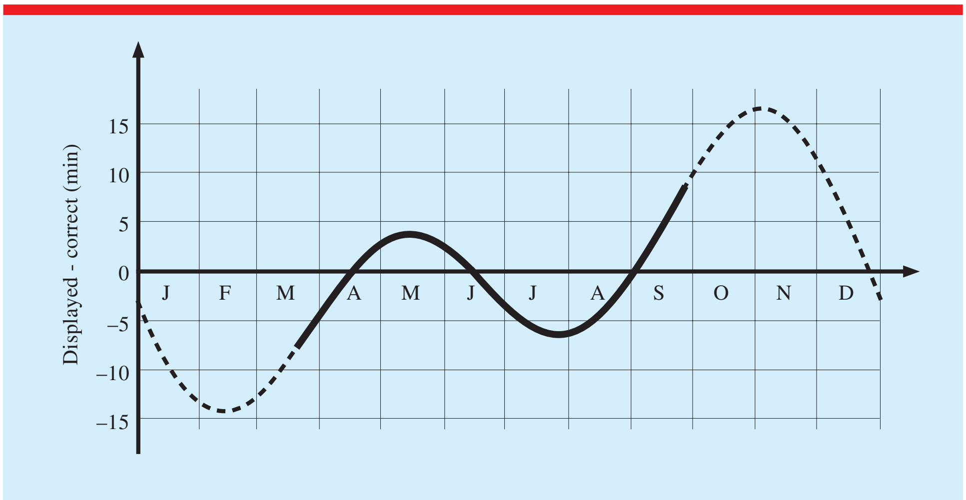

Figure 6. The time equation graph.

11:10 am. From the graph of figure 6 we can see that on 19 May the reading of the sundial is about 4 min more than the correct zone time. We conclude that the exact zone time is approximately 11:06 am. This value indeed agrees, within measurement precision, with the *actual* correct zone time, 11:05 a.m., shown on the wristwatch on photo 5.

Based on the principles explained above, we can now construct our correctly operating, simple CD sundial. A template, designed for a given latitude ϕ and longitude λ, is shown in figure 7.

Instructions: Using the specific latitude ϕ and longitude λ of your location, construct a figure analogous to figure 7, and print it on an A4 sheet of paper. Make sure that the large circle has the same size as a CD or DVD. Glue the printed A4 sheet on a cardboard to increase mechanical stability. Cut it along the two horizontal lines shown in figure 7. Fold it along the five broken lines and make it into the shape shown in photos 4 and 5. Glue the appropriate surfaces together. Be careful to make the two side surfaces, one of which contains the time equation graph in figure 7, exactly vertical. That will set the tilt angle of the CD to $90° - \phi$ with respect to the horizontal, thus by orienting the sundial toward the North the surface of the CD will be parallel to Earth's Equator. Calibrate the dial according to the principles written above: start from a completely symmetrical arrangement of calibration markings with the number 12 (if your country uses DST then with the number 1) at the top of the dial; then rotate the entire scale through the angle by which your latitude λ differs from the latitude of the center of your time zone (the latter can be written as m·15°, an integral multiple of 15°). If the difference is positive, the rotation should be clockwise, if negative, it should be counter-clockwise. For example, for Budapest $\lambda = 19°$ and $m = 1$, so the symmetrical calibration markings must be rotated through an angle $\lambda - m \cdot 15° = +4°$ in the clockwise direction.

A historical aside: The standardization of zone times was not based on rational considerations alone. While Pakistan, like most countries, adopts the rule that zone time should differ from GMT by an integral number of hours, and sets its zone time to GMT + 5, the neighboring country of India uses a zone time of 'GMT + 5 h *30* min' and Nepal sets its zone time to 'GMT + 5 h 45 min'. The proper calibration of the equatorial sundial for such exceptional cases is an enjoyable mental exercise which the reader is invited to do on her own.

5. A transparent CD sundial that can be used throughout the year, with a terrestrial globe as a pedagogical tool

The CD sundial described above is a compact, elegant device, but since CD's are not transparent, it can only be used between March and September. Fortunately, this drawback can be eliminated. One can find several video tutorials on internet on how the non-transparent printed layer can be removed from the surface of a CD in a very simple way. First, cut a tiny line on the printed layer with a

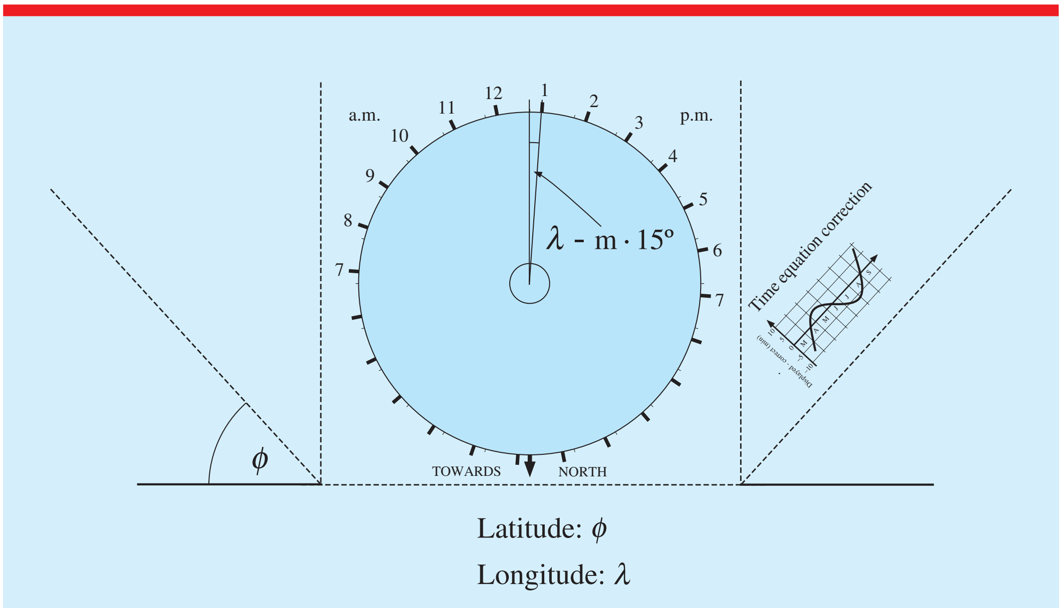

Figure 7. Design template for a correctly operating, simple CD sundial.

Photo 8. Removing the non-transparent layer from the surface of a CD.

knife. Then, starting along this tiny line use sticky tapes to remove the printed layer from the entire CD. The process in illustrated in photo 8.

As a result we obtain a *transparent CD* that acts both as a transmission grating and as a reflection grating. The diffraction efficiency of the transparent CD, i.e. the intensity of the sharp colorful diffraction line, turns out to be about the same in transmission as in reflection. With such a CD installed as the dial, we will be able to use our sundial *throughout the year*. Photo 9 shows such a transparent CD sundial. (Note that if we want to supply our transparent

Photo 9. Transparent CD sundial that can be used throughout the year.

CD sundial with the time equation graph, we have to use the *entire graph* in figure 6.)

Compared with photo 5, it can be seen that the cardboard frame now has holes, so that sunlight can reach the bottom surface of the CD between September and March. Photo 10 shows the same sundial from the appropriate viewing angle. The photo was taken on 6 June. The time displayed on the CD is appr. 1:50 pm (± 3 min for measurement errors). We correct this value using the time equation graph of figure 6 for 6 June: we substract 2 min. The time, according to our sundial, is thus 1:48 pm (± 3 min), in excellent agreement with the exact zone time shown on the wristwatch on the photo.

As seen on photos 9 and 10, to this sundial configuration I added a small terrestrial globe for pedagogical reasons. The terrestrial globe I used is a ball made of foam rubber, available in many map stores. I cut the globe into two halves along the plane of the Equator, and glued the two hemispheres to the upper and lower surfaces of the CD, respectively, paying attention to their proper matching. The small foam rubber terrestrial globe I used is more like a toy than a serious measurement tool; the lines of longitude and lattitude and the country borders are not printed on it with high precision. Luckily, however, it displays lines of longitude in 15° (rather than, say, 10°) intervals. This was ideal for the sundial, since it enabled me to draw the radial hour lines on the surface of the CD as natural 'extensions', when viewed from above, of the lines of longitude on the small globe. This can be seen on photo 10.

The addition of the small terrestrial globe to the CD sundial has great pedagogical value, for several reasons:

1. The small terrestrial globe makes all aspects of designing a sundial visually apparent and easy to grasp. Is the dial of the sundial to be set in parallel with the plane of Earth's Equator? The small terrestrial globe shows this immediately, since the CD *is* the plane of the small globe's Equator. Is the calibrated dial to be rotated through an angle that corresponds to our longitude λ? The small terrestrial globe illustrates this, too: First, we have to rotate the small globe around its North–South axis so that our location, printed on the small globe, is *exactly on the top*. Second, we have to draw the radial hour lines on the CD so that they are extensions, when viewed from the North Pole of the small globe, of the 0°, 15°, 30°, etc. lines of longitude printed on the small globe. Third, we have to write the

Photo 10. The transparent CD sundial as seen from a perpendicular viewpoint.

hour-markings on our calibrated dial so that, from all the lines of longitude printed on the small globe at 15° intervals, the one corresponding to our time zone should point to the '12 o'clock' marking on our dial. (If we want our sundial to display DST too, then the same marking should also say '1 pm DST'.)

All of what is written above is illustrated on photo 10, for a sundial designed for the location of Budapest. The small globe is oriented so that the printed dot for 'Budapest' is exactly at the top. Budapest is in the time zone of GMT + 1, so the radial calibration line along the +15° line of longitude points to the '12 o'clock' ('1 pm DST') marking on the dial. As explained in section 3, this marking is not exactly at the top of the dial, but at a position rotated clockwise by 4° from the top.

2. Terrestrial globes used as desktop objects, lamps, etc. usually have their North–South axis tilted at 23.5° *from the vertical*. In other words, when we look at such a terrestrial globe as a model of Earth, the plane of the table on which the globe rests (or, rather, the plane parallel to the table and going through the center of the globe) is assumed to model the orbital plane of Earth. In contrast, the small terrestrial globe on photos 9 and 10 *has precisely the same orientation in space as Earth itself does*. For example, if our sundial was designed for Budapest (the values of λ and ϕ were chosen accordingly in figure 7) then the tiny printed dot representing Budapest will be exactly at the top of the small globe, and the tangent plane at that point will be horizontal, i.e. parallel to the ground on which the observer is standing while she is looking at the sundial. If she sticks a needle or puts a tiny figure on the small globe at the point representing Budapest, that tiny figure will stand in parallel with her. This phenomenon is illustrated in figure 11. Since the small terrestrial globe has the same orientation in space as the real Earth, the sunlight shining on the small globe at any given moment shows precisely *how sunlight actually illuminates our planet at that same moment*. By looking at the small globe, we can actually see which countries along which line of longitude are just experiencing local noon (it is where the sharp diffraction line on the CD is pointing), in which regions is Sun setting and rising (it is along the two lines that separate light from shadow on the small globe). The line of longitude exactly opposite the sharp diffraction line (in other words, the

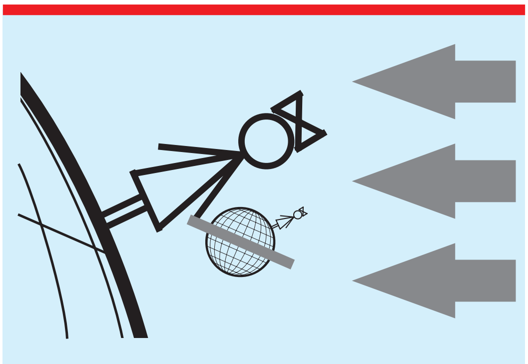

Figure 11. The small terrestrial globe has the same orientation in space as Earth itself does.

middle line of the dark region on the small globe) shows the places on Earth where it is local midnight. By looking at the small globe we can also see the line of latitude above which Sun is not setting at all that day (if the observation is done during summer), or not rising at all that day (if the observation is done during winter). For example, on photo 10, we can read the following information from the sundial, besides the exact time: at the moment when the photo was taken, it was local noon in certain parts of England and Algeria; Sun had just risen in Montréal; Sun had already set in Beijing; and it was a *white night* to the North of Alaska.

3. On the small terrestrial globe we can even draw the lines that separate *time zones*. (These lines usually run along country borders, but sometimes follow strange and surprising patterns.) That way the CD sundial will also illustrate, in a directly accessible visual way, the difference between local time and zone time.

As mentioned above, to read precise time off the CD sundial we have to look at the dial *exactly from a perpendicular direction*. Unfortunately, we cannot use the same trick for the transparent CD sundial of photo 10 that we did for the CD sundial of photo 5, because the transparent CD does not reflect our face in an efficient way (especially if we also glued a small terrestrial globe to its center). One way to get around this problem may be to orient our line of view so that the great circles of longitude look straight radial lines, and the circles of latitude look concentric circles (see photo 10). This, however, is difficult to do with high precision. A more precise solution is to stick a small needle radially into the globe at the North Pole. Proper viewing orientation is then achieved by looking at the globe with one eye open, and making sure that the needle appears to be a single point.

6. Concluding remarks

By examining photo 10 we can make an interesting discovery. *The small terrestrial globe is, in itself, a sundial!* The small bright spot seen on the surface of the globe is the reflected image of the Sun, and it works just like the sharp radial diffraction line on the CD. They point in the same direction, so the bright spot on the globe's surface can also be used to display the correct time on the sundial, although with less precision than the sharp diffraction line. (Note, however, that unless we use a transparent globe, for this to work we need the bright spot to appear on the Northern hemisphere of the globe, and that only happens in the half year between March and September.) If we are willing to trade in some measurement precision for a simpler, more compact and pedagogically clearer design, then we may just as well get rid of the CD entirely. Our sundial will then consist of a single small terrestrial globe, carefully oriented so that its orientation in space exactly matches that of Earth (see figure 11). Its 'dial' will be the small globe itself, with the hour-markings printed on lines of longitude separated by 15° intervals. This is perhaps, in terms of pedagogy, the simplest, most clear-cut of all sundial designs: we take 'God's viewpoint' of our Earth from the Pole Star, and make a small-scale model of it. In 'God's viewpoint' the dial is the Earth (in our case it is the properly oriented small terrestrial globe), and the hour hand is Sun's glistening reflection on Earth (in our case Sun's reflection on the small globe).

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