Massimo Goretti

The linear astrolabe of al-Tusi

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The linear astrolabe of al-Tusi gnomonic applications

This article summarizes the results of historical research on the linear astrolabe of al-Tusi, examining the construction and methods of employment of this medieval astronomical instrument of Islamic origin.

Massimo Goretti

Sharaf al-Din al-Muzzaffar ibn Muzaffar al-Tusi was born in 1156 AD in Tus, in the region of Khorassan , circa 1000 Km from Teheran in north-western Iran, near the Afghan border. He died in Baghdad in 1242. In the Islamic world, from the point of view of Astronomy, he is considered so important that his contemporaries compared him to Ptolemy.

His astrolabe is described in the "Traité des instruments" by Ab'u – l'Hassan Ali.
The chapter related to this astrolabe was translated in 1895 by Baron Carrà de Vaux in the "Journal Asiatique" with the title "L'astrolabe linéaire ou baton d'Al-Tousi.
Successively, in 1947 H. Michel addressed the subject in chapters 14 & 15 of the book "Traité de l'astrolabe" and constructed a model of the device which is housed at the Oxford Museum of Science.
In, 1999 R.D'Hollander discussed the instrument in chapter 6 of his treatise "L'Astrolabe – Histoire,théorie et pratique "
In 2007, J. E. Morrison, in his book "The astrolabe" da pag.287 addresses the question from pg. 287 to 292 with an original approach, different from his predecessors.
In these chronicles, of interest is the prologue of the book by Baron Carra de Vaux which addresses, for the first time, this instrument and the interest it aroused. Here are the essential parts of the text. .
In the "Mèmoire sur les instruments astronomiques des arabes" by L.A. Sedillot, the author in various points speaks of the "astrolabe linéare ou baguatte de Nasir ed Din Tousi", however, nowhere in the text can be found a description of the device. Sedillot promises to prepare a second work on Nasir de Din, in which the author will elaborate on the staff of Tousi. However, this additional work by Sedillot never appeared; furthermore, there existed the notable suspicion that this instrument and the staff of Jacob were in fact the same thing.
Clarity on this topic was deferred to the translation of the Arabic manuscript undertaken by the Baron Carra de Vaux
In the translation, the first thing which emerged was that Sédillot had committed an error in attributing the staff to the famous Nasir-ed Din al-Tusi; the real inventor was instead Sharaf al-Din al-Muzzaffar ibn Muzaffar al-Tusi, a contemporary of Kemal ed Din Ibn Yunis who lived from 1156 to 1242.

In the first part of the manuscript one finds the device's description, which by itself would have been sufficient in making understood that the staff is a very special kind of astrolabe. In the second part, its use is explained. Lacking in the manuscript are any drawings or designs of the instrument.
The essential results of this study can be summarized as follows:
1) The linear astrolabe is a true astrolabe; it derives from the planispheric type and, like the latter, is essentially a plane on which the celestial sphere and the various circles of declination are projected; on a particular line of this plane the projection of these points takes place. 2) In practice, the instrument resembles a slide ruler in appearance and in application. 3) The measurement of angles is done via lines added to the staff: These give the lengths of the cords stretched from the arcs, then, through gradations traced on the rim, the corresponding angles are created.

The building of this astrolabe, in the initial stages, makes reference to the classic construction of planispheric astrolabes, utilizing the principal of stereographic projection in which the observer is located at the South Pole. (Fig.1)

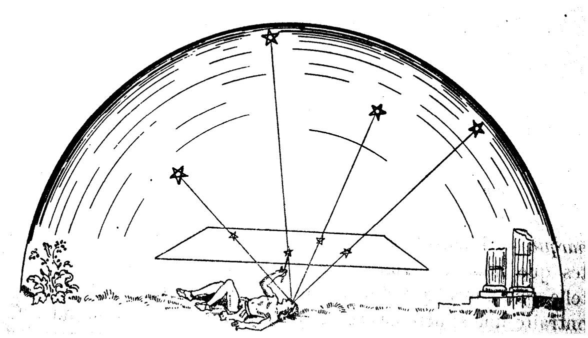


Fig.1

This system has the following limitations::
1) The astrolabe is only valid for latitudes close that of construction (in this case a latitude of $\varphi=43°$ has been hypothesized).
2) The astrolabe is constructed in a certain date, hence the sky is "photographed" in a certain moment. With constant variation over time of the right ascension and the declination of the celestial bodies, an error is made related to the difference between the date of readings and date of construction. With such a device, an approximate validity of 50 years is supposed.

THE PARTS OF THE ASTROLABE

To understand the characteristics and limits of this instrument it's useful to reference a simplified version of the polar astrolabe. In particular, as we shall see, the tympan of the astrolabe should be considered as reduced with respect to the path of the almucatarat and the rete is accordingly simplified with the ecliptic and with the indication of a limited number of stars, among those closest as possible to the circle of the Equinox, correctly distributed along it, so that each night it is possible to see at least one of these stars.

Circles of declination

1) Trace a circle with center and radius OF that represents the tropic of Capricorn (FGHI)
2) From F→ I create an angle equal to the inclination of the ecliptic and identify point X.
3) Trace XG, which encounters the vertical at K. Trace the circle with radius OK. This circle is the equator.
4) Join X with O and identify point T on the equator, trace TL which encounters the vertical at M. The circle with radius 0M represents the tropic of Cancer.
The trigonometric expression of the circle of declination is summarized in:

$$R = Re\ tg\ (45 \pm \delta/2)$$

Re= R♈= R♎ radius of the equator
δ declination of celestial body

Substituting at δ the corresponding values of the declination of the Sun at the various signs of the Zodiac, the radius (plural) of the circles of declination: (Fig.2).

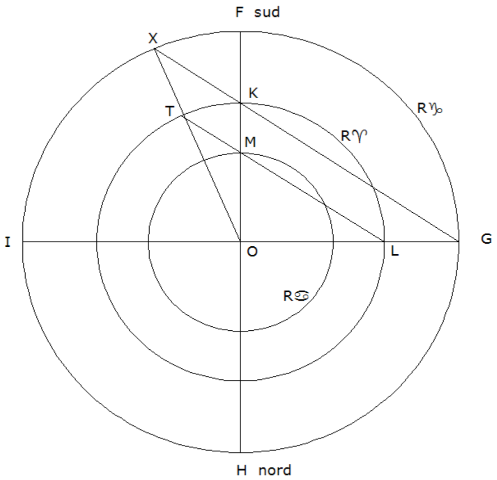


Fig.2

Circles of Altitude (almucatarat)

In the figure below there is a method for the graphic identification of the almucatarat on the tympan of the astrolabe. The first ring represents the local horizon.

Working through trigonometrics, taking point 0 as origin, the ordinates of the centers of the almucatarats are given:

$C_i = R_e \cos\varphi / (\sin\varphi + \sin h)$ h= 0°-10°….90°

The ordinates of the passage of the almucatarats on the meridian circle, towards north, are given by:

$P_i = - R_e \, tg[(\varphi - h)/2]$ h= 0°-10°….90°

$R_e = R♈ = R♎$ radius of the equator

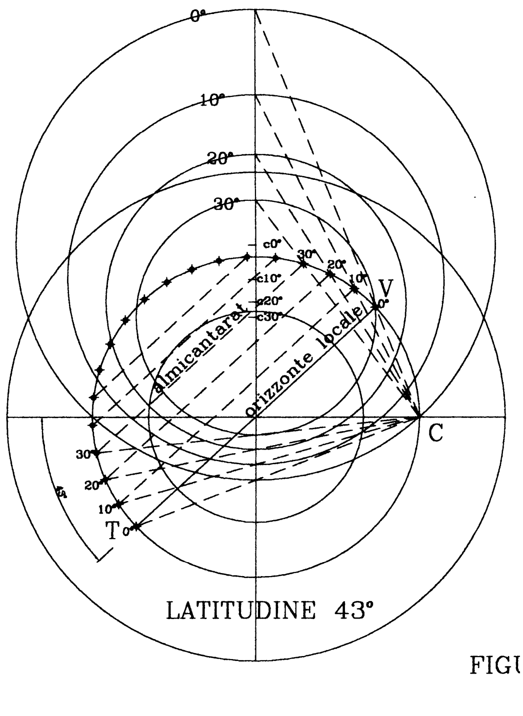
φ latitude of location

h altitude of celestial body

Observe that C_0 represents the horizon and C_{90} the zenith.

(Fig.3)

Fig.3



In the figure below, simplified, there is the tympan of the astrolabe, on which have been traced several circles of declination of the Sun and the almucatarat.

If we make a section in the meridian plane, we obtain the astrolabe di al-Tusi. This is shown in the right part of the figure. (Fig.4).

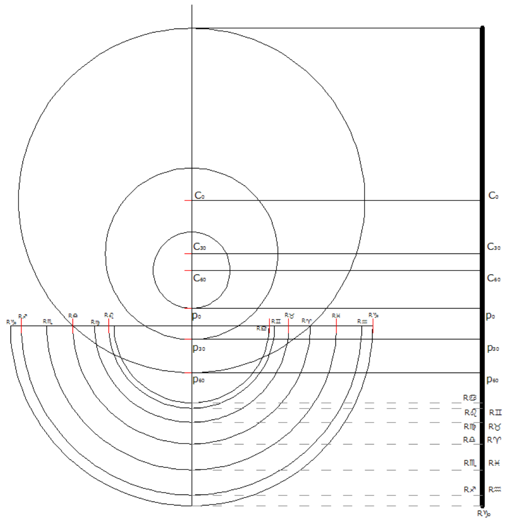


fig.4

Scale of the cords

The next step is to transfer onto the staff the functions explicated, on the astrolabe, by the slide ruler and graduated scale.
One obstacle to overcome is the measurement of the angles. Here Al Tusi demonstrates his genius.
The medieval Islamic world knew the function of the "cord": given the length of a cord, it was possible to find the value of the corresponding angle. (Fig.5).
$Crd\ \alpha = 2R\sin\alpha/2$
The scale of the cords may be constructed with
$aa1 = 2\ R♑ \sin\alpha/2$
where
R♑ radius of the circle of declination of Capricorn.

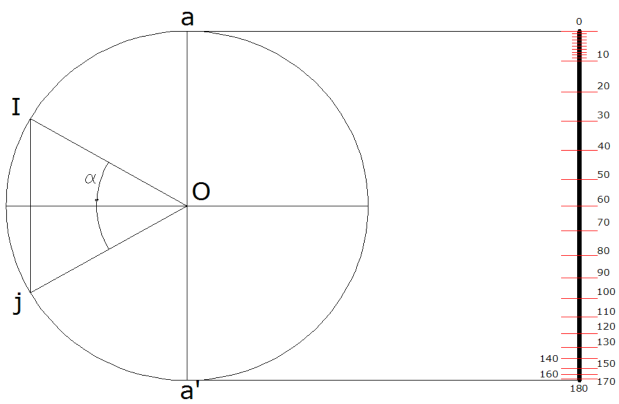
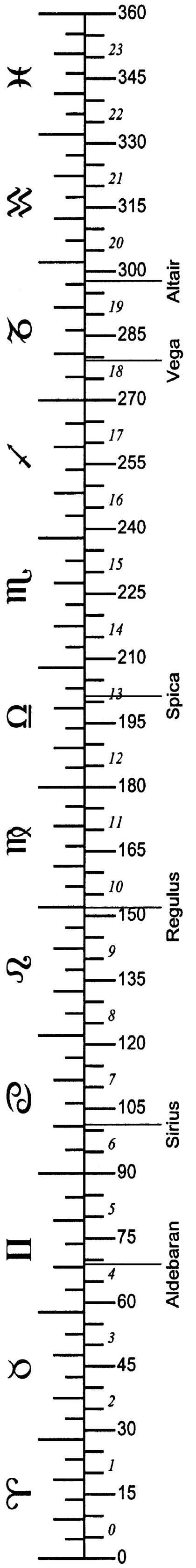


Fig.5

Scale of right ascensions – Circles of declination of the stars



The astrolabe is also used at night to determine, through the measurement of the height of the celestial body, the corresponding hour.

To reach this measurement through the relation

$Ts = t_* + (AR_* - ARs)$ $\quad t_*$ star time

At the time of the Sun it's necessary to construct an arbitrary scale which contains the right ascension of the Sun ARs during the course of the year and the corresponding ascension of the star AR_*.

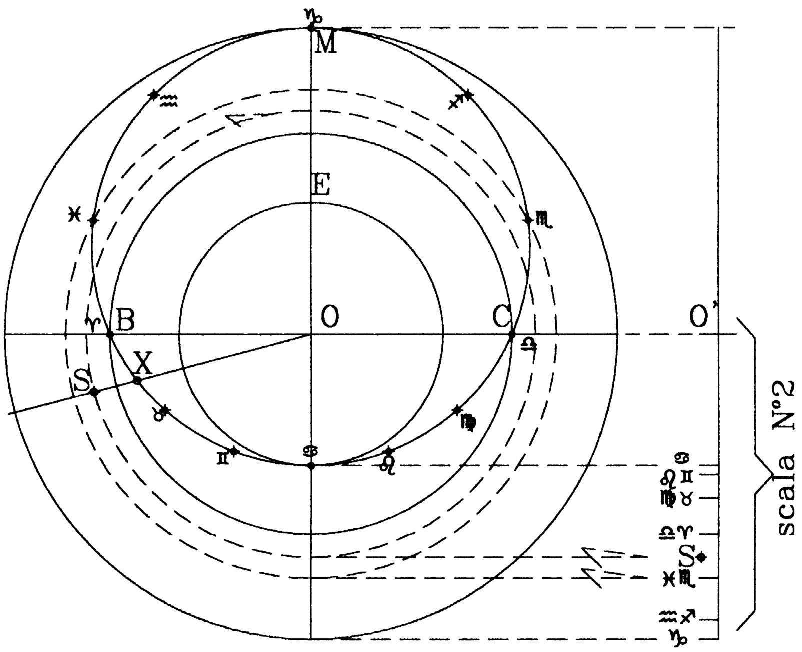
Figure 6 depicts this scale, indicating the position of several stars. In terms of the position of the stars, if we make reference to the essential lines of the rete of a polar astrolabe, we observe that, imagining a rotation of the ecliptic around the center O, each of the points of the ecliptic trace a circle. Any possible stars S existing in the rete travel in a circle with radius OS. As an example, we've traced the circles traveled along from the beginning of Pisces (and its symetric Scorpius)and the star S.

The figure below clearly explains the scale of declinations possessed by the instrument O: O' is at the center of the instrument and the scale indicates the distances O'♋,O'♌,O'♍, etc …, in other words, the radii relative to the point of origin of each of the zodiac signs. Star S (indicated as an example) is described on the scale 2 in order to reconstruct its own circle s. Its right ascension AR_* is identified by the point X of the ecliptic.

Trigonometrically speaking, the radius of declination of the star is always given by

$R_* = Re\ tg(45 \pm \delta/2)$

δ declinazione astro



Lines - Pinnules

The peculiar aspect of this astrolabe is its three lines. (Fig.8).

Fig.8

Line 1 – Plumb line fixed in the center O

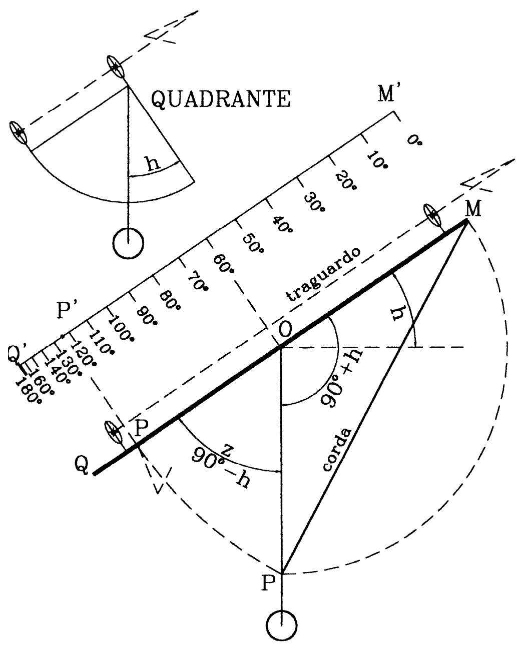
Line 2 - Measurement line fixed at the southern extremity.

Line 3 - Adjustable tension line.

There are also two pinnules which function to sight and focus the celestial body to be observed.

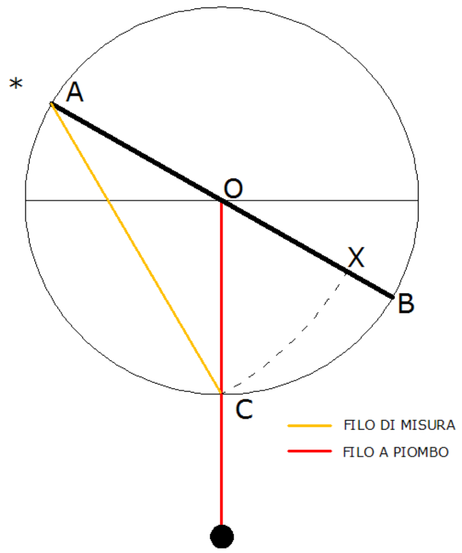
USES OF THE ASTROLABE

Here are descriptions of several operations that can be conducted with the astrolabe.



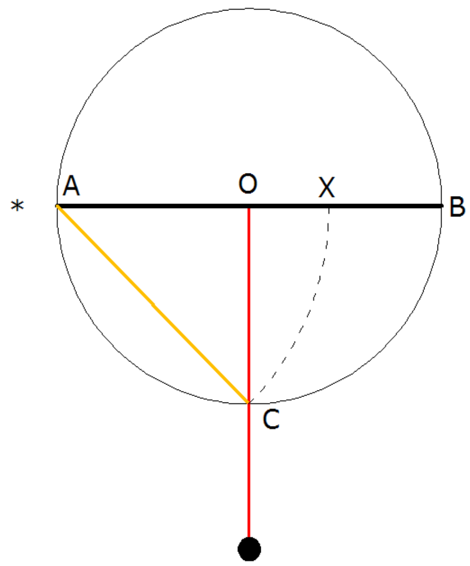
The setup of the instrument is very similar to a quadrant of altitude (compare the two in the small figure here present), and both are used in similar ways. The height h of the celestial body (or the zenith angle z) is determined and the scale of the cords enables one to substitute the goniometer in the measure of the angle MOP.

Heigth of a Celestial Body



1) The celestial body is sighted through the pinnules
2) The length of the measurement line [filo di misura] is adjusted until the plumb line [filo a piombo] is vertical
3) The length of the measurement line is transferred to the scale of the cords and the angle is read.. AOC. The string AC is the measure of the angle AOC= (90+h).
The height h of the celestial body is given by
h= AOC-90°
Knowing the maximum height H of the Sun
H= 90-δ-φ
It's possible to arrive at the temporary hour T
Sin15T=sinh/sinH

At the moment of rising of a celestial body, following the same steps, it's possible to obtain,



AX=90
h= 90°-90°=O°

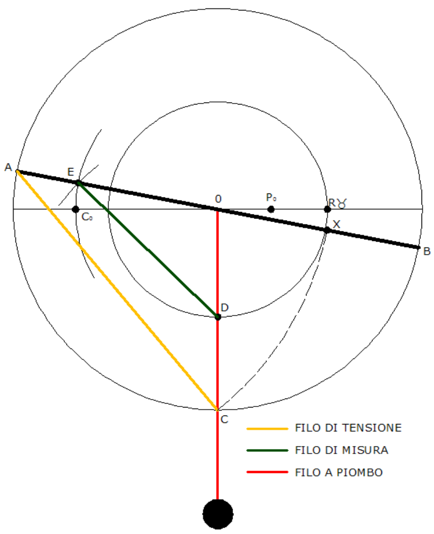
Semidiurnal Arc

The semidiurnal arc is the amplitude of the arc traveled by the Sun from rising to the meridian. Let's say we want to determine the semidiurnal arc for April 21 (beginning of Taurus)

1) Set plumb line at a distance of OD=OR♉
2) Adjust tension line to length of ED=C_op_o and affix it to the plumb line at D and C_o
3) Rotate the staff, varying the length of the measurement line such that the plumb line remains vertical.
4) The semidiurnal arc is the angle EOD, the measurement line AC on the scale of the cords does nothing less than define this angle. Consider this as ex.: AX= 100°

The Sun rises
ts= 12-(100/15)= 5,34 = **5h20min**
The semidiurnal arc is
ω= 12-5.34= 6.66 = 6h40min
The sun sets
t= 12+6h40min= 18h40min
The length of the day
Tg=2 ω = 2x6.66= 13.32= **13h20min**

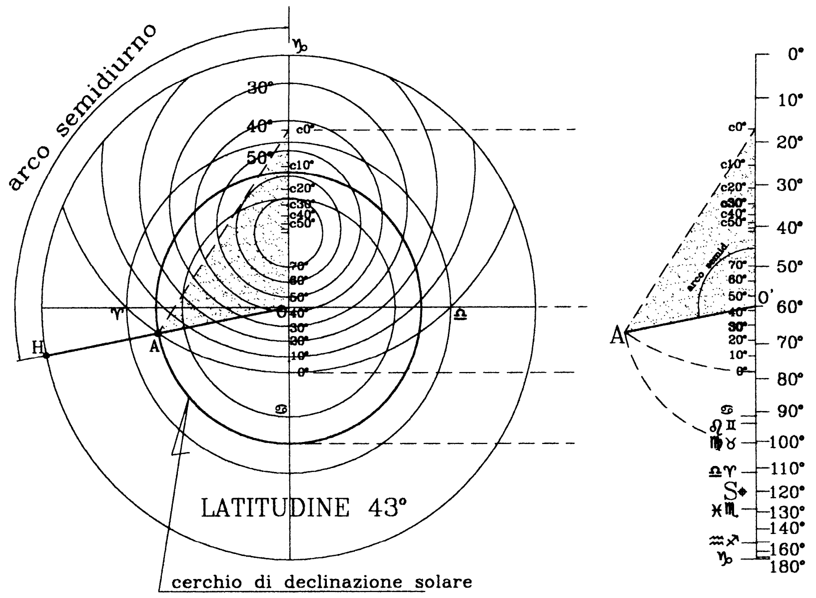
Observe that since C_0 is the center of the almuncatar with a height 0° and R♉ the radius of declination of Taurus, the method resolves the problem of determining the point of intersection between circles of altitude and circles of declination.



OD=OR♉ DE=CoPo **AX= 100**

The justification of the above statements is given by observing what happens in the astrolabe: the generic semidiurnal arc is that represented in the shaded space between OA (radius of declination of the celestial body) and OCo (radius of the almucatarat with height h=0). In the astrolabe of al-Tusi it's this angle which must be reconstructed via the tension line, while the
measurement line, transferred onto the scale of the
cords, determines the angle's value. (Fig.10-11)

Fig. 10



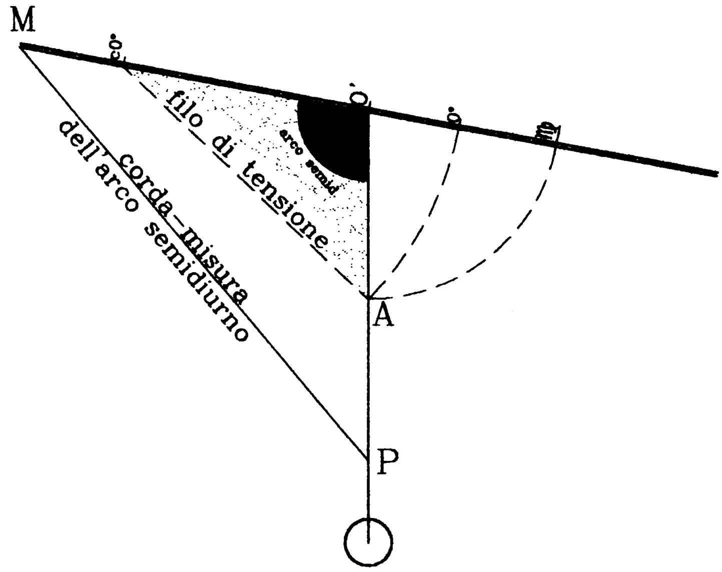


Fig.11

Determining Time with the Sun.

On April 21 the sun's height is measured at h=30°, how does one determine the time of measurement? (Fig.12)

1) Set the plumb line at a distance OR♉

2) Adjust the tension line to $C_{30}p_{30}$ and affix on the plumb line and at C_{30}

3) Rotate the staff, varying the length of the measurement line such that the plumb line remains vertical. In this way the angle of time AOD is defined.

4) This length AC (which is the cord of the angle AOD) is transferred to the scale of the cords and the value of the corresponding angle is read. Ex. AX= 58°
The time of measurement is:
t= 12-(58/15) = 8.13= **8h8min**

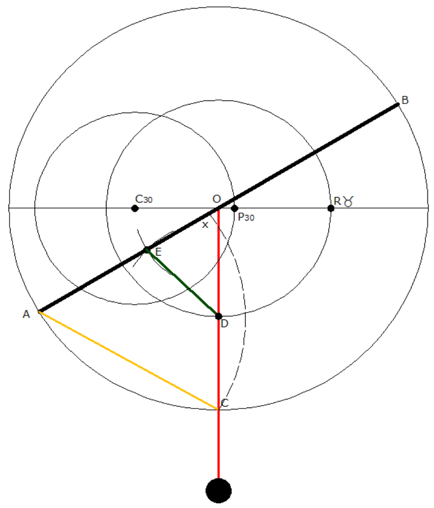


Fig.12

OD=OR♉ DE=$C_{30}P_{30}$ **AX =58**

Another example:
On December 21 the Sun's height is measured at h=20°, how does one determine the time of measurement? (Fig.13)

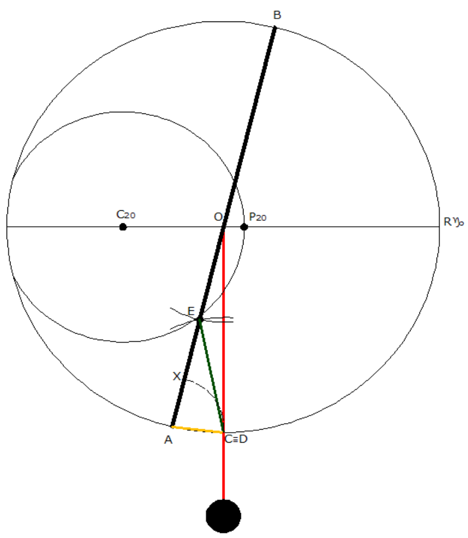


Fig.13

OD=OR♑ DE=$C_{20}P_{20}$ **AX = 115**
1) Set the plumb line to a distance of OR♑
2) Adjust the measurement line to the length $C_{20}p_{20}$ and affix on the plumb line and at C_{20}
3) Rotate the staff, varying the length of the measurement line such that the plumb line remains vertical. In this way the angle of time AOD is defined.
4) This length AC is transferred to the scale of the cords and the value is read: Ex. AX= 115°
The time of measurement is:
t= 12-(115-90/15) = 10.33= **10h19min**

Determining time at night with the stars

The operation is the same as that described for the determining time during the day, but, it must be executed via the star S. On the Astrolabe the point of the star is placed on the almucatarat as found. (in the example: 20°) The triangle thus obtained is SOC20°, and the design of the staff of al-Tusi is presented in In this case, however, we have the time of the Star with respect to its culmination.
On the polar astrolabe, supposing that the Sun is at the
beginning of Libra, it's easy to find the corresponding time of night time by reading at point K.
To find the time with the staff of al-Tusi, it's necessary to know the difference between the right ascension of the Sun on that day and the right ascension of the star. To this end, on the staff a table of right ascensions is constructed.
Thus it's easy to find the time gap between the star and the Sun, and it's possible to reconstruct at what time of night the star is in that point. As an approximate example, consider the Sun in Libra with a right ascension of 180°, while the Right Ascension of the Star is 15°: the difference is 180 - 15 = 165°, corresponding to 11 hours; this will be subtracted from the hour of the Star to obtain the local time. (Fig.14)

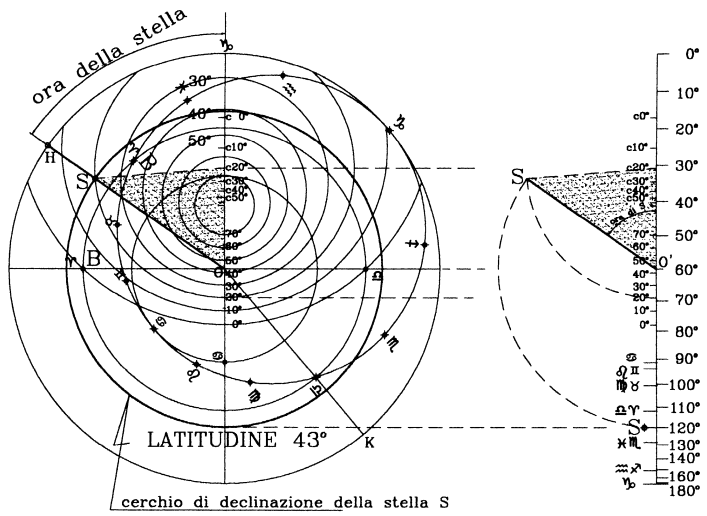
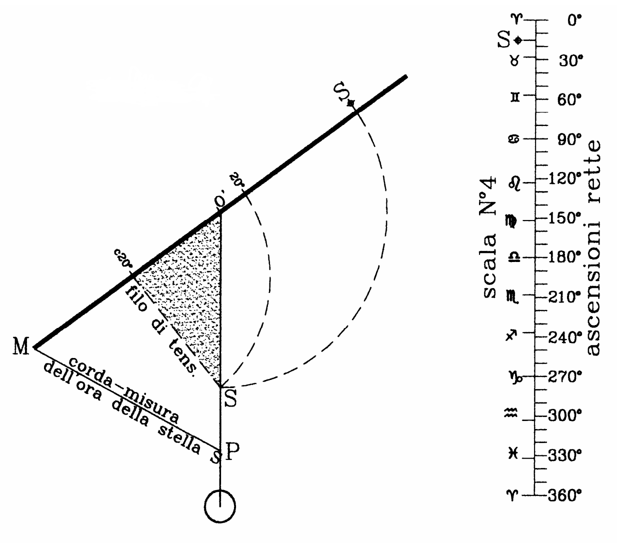


Fig.14



For example, on April 21 the altitude of Spica is measured as h=30°; how to determine the time of measurement? (Fig.15)
The time of the star is given by the angle AOD

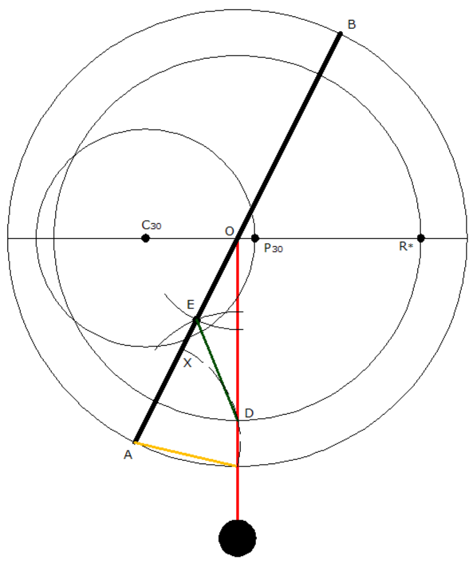


Fig..15

OD=OR* DE= $C_{30}p_{30}$ **AX= 27**

The hour of the Sun is given as:
Ts= [h+(AR*-ARs)]
AR* right ascension of the star AR_*= 201°25'
ARs right ascension of the Sun ARs= 29°20'
ΔAR= 172°5'=11,47 h
Hour angle of the measured star

1) Set the plumb line to a distance of OR*
2) Adjust the tension line to a length of $C_{30}p_{30}$ and affix to the plumb line and at C_{30}
3) Rotate the staff, varying the length of the measurement line such that the plumb line remains vertical.
4) This length AC, that represents the cord of the angle AOD, is transferred to the scale of the cords and the value is read Es. AX= 27°
The time of measurement is:
t= 180-27/15+11,47=21.67= **21h40min**

Observe that, if on the staff we construct on the plumb line the distance OD, equal to the radius of declination of the star, and on the tension line the length equal to the radius of the almucatarat relative to the measured height, the angle AOD is the time of the star and the length AC of the measurement line supplies the value of this angle.

To further clarify: On June 21, the height of Altair is measured as h=30°; how to determine the time of measurement? (Fig.16)
1) Set the plumb line to a distance of OR*=OD
2) Adjust the tension line to a length of $C_{30}p_{30}$ and affix it to the plumb line and at C_{30}
3) Rotate the staff, varying the length of the measurement line AC such that the plumb line remains vertical.
4) This length AC, which represents the cord of the angle AOD, is transferred to the scale of the cords and the value is read Es. AX= 52°

AR* right ascension of the star AR_{*}= 19,51h
ARs right ascension of the Sun ARs=6h

ΔAR=13,51 h
The time of measurement is:
t=(180-52/15)+13.51 =22,04= **22h2min**

OD=OR*= DE= $C_{30}p_{30}$ **AX= 52**

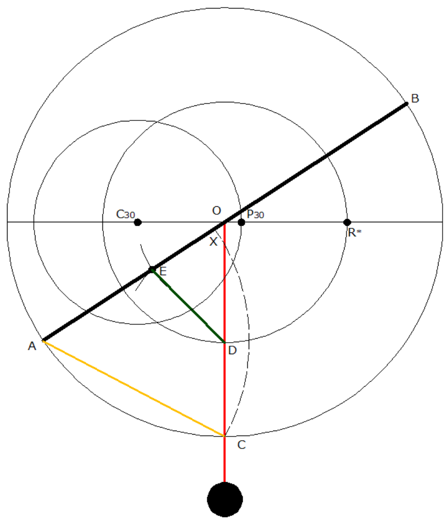


Fig.16

It's evident that the staff functions like a slide ruler which can resolve the problems correlating to the intersection of the circles of altitude and circles of declination of a celestial body.

The examples cited above demonstrate the genius of this simple instrument, with its ease of construction, use of common materials, and ease of application. Furthermore, the device permits grand and costly astrolabes to be substituted by an ordinary staff. On the negative side, there is the somewhat laborious method of use and scant precision; furthermore, application must be supported by a good knowledge of astronomy and mathematics.

The historical research that accompanied the construction of this instrument of astronomy and of calculation confirmed for me the vast astronomical knowledge and intellectual capacity of the Medieval World (especially the Islamic world). I maintain that the definition of "Dark Ages" that has accompanied this historical era is only a mix of rhetoric and prejudice.

I would like to conclude, with a touch of regret, with the words H. Michel (Ciel et Terre 1943) "*most likely the staff did not add anything precious to the collections of astrolabes and could not justify the honor of their display cases. For this reason the staffs have not been preserved. Unfortunately, this is the fate of those instruments which are too modest.*"

I wish to thank my friend Alessandro Gunella and Jim Morrison for their valuable and decisive suggestions.

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